Contents

1 Bass stable range 1
1.1 Stable range conditions 1
1.2 Ring theoretical constructions and their stable range 10
1.3 Completion to invertible matrix 10
1.4 Stabilization in K-theory 10
1.5 Examples and stable range calculations 10

2 Rings ruled by units, annihilators and idempotents 11
2.1 Von Neumann regular rings 11
2.2 Exchange, clean and idempotent stable range one rings 11
2.3 Potent and semipotent rings 21
2.4 VNL, NJ and semiregular rings 21
2.5 PM and Gelfand rings 21
2.6 Semihereditary and morphic rings 21

3 Rings defined by matrix canonical forms 24
3.1 Hermite and elementary divisor rings 24
3.2 Bezout rings and Shores test 30
3.3 Stable range and matrix diagonalization 30
3.4 Pullbacks and D+M-construction 30

4 Adequacy 34
4.1 Adequate and coadequate elements 34
4.2 Stable range of adequate rings 40
4.3 Zero-adequate and everywhere adequate rings 42

5 Finite homomorphic images 48
5.1 Minimal prime spectrum and fractionally regular rings 48

5.2 Semiregularity of zero-adequate rings 55
5.3 Avoidable rings 57
5.4 Effective and Dirichlet rings 60
5.5 Neat range one 63
5.6 Meaningful ring 65
5.7 Bezout morphic rings and units lifting 71
5.8 Gelfand range one and Bezout PM*-domains 75
5.9 Lattice-ordered groups and Montgomery counterexample 79
5.10 Rings of continuous functions C(X) 83

6 Bezout domains and their overrings 85
6.1 Almost stable range one 85
6.2 Finite localizing embeddings 92
6.3 Full matrices over elementary divisor rings 99
6.4 Sharp Bezout domains 103

7 Diagonalization over noncommutative rings 107
7.1 Simple Ore and Bezout rings 107
7.2 Idempotent matrix diagonalization 119
7.3 Distributive Bezout rings and Dubrovin condition 121
7.4 When $GL_n(R)$ is closed under transposition 123
7.5 Right Bezout rings and unimodular rows 126

8 Rings defined by range conditions 129
8.1 Neat range one and stable range 129
8.2 Semihereditary and von Neumann regular range one 130
8.3 Additive range one 137
8.4 Dyadic range one 137

9 Problems related to range conditions 143
9.1 Rings ruled by annihilators and idempotents 144
9.2 Commutative Bezout Rings 146
9.3 Nontrivial Finite Homomorphic Images of Commutative Bezout Rings 146
9.4 Adequate Rings and Their Generalizations 147
9.5 Local properties and ranges of commutative rings 148
9.6 Noncommutative Bezout rings 150
9.7 Ultimate problems 152